TODAY (1) χ^2 DISTRIBUTION "WHAT IS IT?"

(2) CONNECTION OF χ^2 STATISTIC W/ DATA

(a) χ^2 DIST'N

(b) POISSON + NORMAL

(c) MERGE OF INDEP χ^2

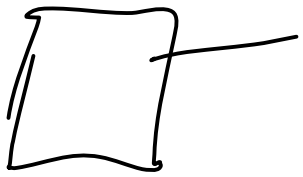
(3) EXAMPLE IN GROUPS. [PICK A PERSON
[ONE IS FIRST GUESSER
[OTHER SECOND GUESSER

(1) MATH DEF OF χ^2 = DIST'N OF $Z_1^2 + \cdots + Z_d^2$

WHERE d = DF, $Z_1, Z_2 \ldots, Z_d$ ARE INDEP STD NORMAL r.v.

(2) $\chi^2_{STAT} = \sum_{\substack{ALL \\ CELLS}} \frac{(obs-exp)^2}{exp} \sim \chi^2_{1\,DF}$

THINK SUBJECTS $1 \to n$ TOSSED INTO CELLS.



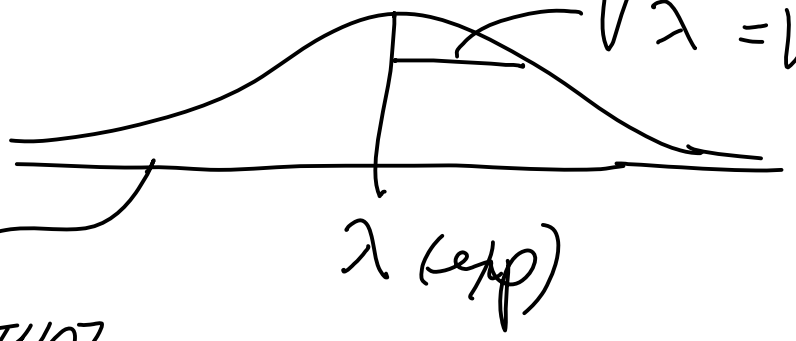
A CELL

POISSON? λ (EXP COUNT) ≥ 3

THEN OBS COUNT $\tilde{\sim}$ DIST

RULE OF THUMB.

$\sqrt{\lambda} = \sqrt{exp}$



λ (exp)

CH 26 WANT exp ≥ 5

SAYING THAT $\frac{obs-exp}{\sqrt{exp}} \sim Z$

SO χ^2_{STAT} HAS AN $\approx \chi^2_{DIST}$

DF = NOT TOTAL # CELLS BUT APPLICABLE DF (CONTEXT)

each cell $\frac{(\text{obs}-\text{exp})^2}{\text{exp}} \sim \chi^2$

$\frac{\text{obs}-\text{exp}}{\sqrt{\text{exp}}} \sim Z$

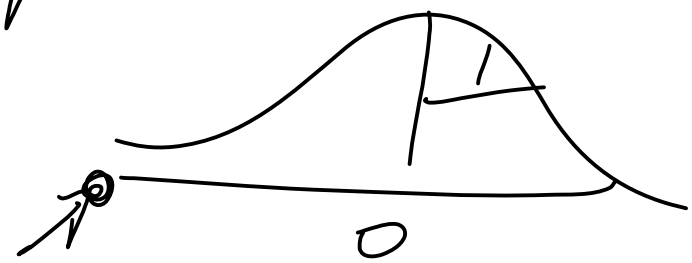
IF H_0 IS CORRECT!

TEXT (CH 26): IF (SAY) P-VALUE $< .0001$

YOU SHOULD REJECT H_0 AND THEN EXAMINE

THOSE CELLS WITH $\frac{\text{obs}-\text{exp}}{\sqrt{\text{exp}}}$ (like Z if H_0 true)

THAT ARE FAR FROM 0.



POSSIBLE $\frac{\text{obs}-\text{exp}}{\sqrt{\text{exp}}}$

(3) RUN THROUGH OF TESTS ASSOC WITH "SECOND GUESSING."

WE HAD MODEL UNDER WHICH

$P(2^{nd}\text{ GUESSER IS CORRECT}) \approx 3/4$

	2^{nd} CORRECT	2^{nd} INCORRECT	
FIRST LOW	2 (2.5)	5 (4.5 = 7 − 2.5)	7
FIRST HIGH	5 (4.5)	8 (8.5)	13
	7	13	20 pro.

Exp: $7 \cdot 7/20 = 2.5$ $\quad \frac{13 \cdot 7}{20} = 4.5$ $\quad 7 \cdot 13/20$

$\chi^2_{STAT} = .225$ TOO CLOSE!

χ^2 TEST OF INDEPENDENCE

LOOK OF INDEP IS PROPORTIONALITY

$DF = (R-1)(C-1) = 1$

$$\left.\begin{aligned}&\frac{(2-2.5)^2}{2.5} + \frac{(5-4.5)^2}{4.5} \\ &+ \frac{(5-4.5)^2}{4.5} + \frac{(8-8.5)^2}{8.5}\end{aligned}\right\} .1 + .05 + .05 + .025$$

GOODNESS OF FIT:

2ND CORRECT

obs	7	13	TOTAL 20
exp	20(3/4)	20(1/4)	UNDER MODEL
	15	5	

(all ≥ 1)

DF = (C − 1)

$$\chi^2_{STAT} = \sum \frac{(OBS - exp)^2}{exp} = \frac{(7-15)^2}{15} + \frac{(13-5)^2}{5}$$

$$= \frac{64}{15} + \frac{64}{15} \text{ again} = 8.5$$

OBS COUNTS	7	13
expected if H_0 TRUE	15	5

χ^2 DF

1

SO P-VALUE < .005